Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16
(Autonomous)
End Semester Examination: Mar/Apr 2025
Faculty: Science and Technology

Program: BScGen03 **Semester: VI** **SET : A**
Program (Specific): BSc(Mathematics) **Course Type: core**
Class: TYBSc(Mathematics) **Max.Marks: 35**
Name of the Course: Partial Differential Equations **Course Code: 24-MT-364**
Time: 2Hrs **Paper: IV**
Instructions to the candidate:

1) ***There are 3 sections in the question paper. Write each section on separate page.***
2) ***All Sections are compulsory.***
3) ***Figures to the right indicate full marks.***
4) ***Draw a well labelled diagram wherever necessary.***

SECTION: A

Q1) Answer the following (Attempt any 5 out of 7) **10 Marks**

1) Find a partial differential equation by eliminating the arbitrary function 'f' from $z = f(x^2 - y^2)$.
2) Define non-linear equation. Also, give one example.
3) Test the equation $e^y dx - (xe^y + 2y)\, dy = 0$ for exactness and solve it if it is exact.
4) Classify the following equation into hyperbolic, parabolic or elliptic type

$$U_{xx} + x^2 U_{yy} = 0$$

5) Find the complementary function of the differential equation

$$\left(D^2 - a^2 D'^2\right)u = 0.$$

6) If u_{CF} and u_{PI} are respectively the complementary function and particular integral of the partial differential equation $F\left(D, D'\right)u = f(x, y)$, then prove that their sum $u_{CF}(x, y) + u_{PI}(x, y)$ is a general solution of given partial differential equation.
7) If $u = x + y$ and $v = x - y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

SECTION: B

Q2) Answer the following (Attempt any 3 out of 5) **15 Marks**

1) Show that the equation z = px + qy is compatible with any equation $f(x, y, z, p, q) = 0$ which is homogeneous in x, y, z.

2) Find the solution of Laplace equation $u_{xx} + u_{yy} = 0$ by using the Separation of variables method.

3) Find the Particular Integral of the following differential equations:

a) $(D - D' - 1)(D - D' - 2)u = e^{2x-y}$

b) $(D - D'^2)u = \cos(x - 3y)$.

4) Prove: $\frac{1}{D-mD'} f(x, y) = \int f(x, c - mx)dx$.

5) Solve the differential equation $(DD' + D - D' - 1)u = xy$ by expanding the particular integral in ascending powers of D and D'.

SECTION: C

Q3) Answer the following (Attempt any 1 out of 2) 10 Marks

1) a) Prove that the necessary condition for integrability of Pfaffian differential equation $P\,dx + Qdy + Rdz = 0$ is

$$P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0. \quad [8\ M]$$

b) If $u_1, u_2 \ldots u_n$ are the solutions of homogeneous partial differential equation $F(D, D')u = 0$, then show that $\sum_{i=1}^{n} C_i u_i$ is also a solution where C_i are constants. [2 M]

2) a) Reduce the equation $U_{xx} + 2U_{xy} + U_{yy} = 0$ to a canonical form. [8 M]

b) Solve the following differential equation by using Lagrange's method.

$zp = -x$. [2 M]